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3-D NUMERICAL STUDY ON THE BENDING OF SYMMETRIC COMPOSITE LAMINATES

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Abstract

In this paper, the composite laminates subjected to pure bending are modelled by using 3-D anisotropic finite elements and the distributions of stress and strain along the thickness are presented. The numerical results show that there is a stress concentration near the bonding interface in the central region (far from traction boundaries and free edges) of symmetric angle-ply laminates, which should be called "interlaminar effect". The abrupt change of the mechanical behavior of material on the interfaces causes interlaminar stresses. The laminates are in 3-D stress state near the interfaces. The strain distribution of the 3-D numerical model is different from the strain hypothesis of the classical laminate theory based on the Kirchhoff-Love hypotheses of straight inextensional normals.

Key words composite, laminate, plate stress analysis, finite element method, three dimension, interlaminar surface, stress concentration

I. Introduction

The expanded use of composite laminates in aircraft, automaton, shipbuilding, and other industries has stimulated interest in the accurate prediction of the response characteristics of composite laminates. Most of the advanced composite laminates in use to date exhibit much lower strength in the transverse direction, thus being particularly susceptible to the interlaminar stresses. Because of the mismatch of Poisson's ratio between adjacent layers, the interfaces gives rise to nonzero interlaminar stresses in bonded laminates. Furthermore, there is a stress concentration near bonding interfaces in the central region of symmetric composite laminates, which should be called as "interlaminar effect". So the interlaminar effect plays a much more important role in predicting the failure in multilayered composite structures.

Several approaches[1] have been proposed to account for the transverse shear flexibility and other nonclassical factors which are neglected in the classical laminate theory[2] (such as transverse normal strain). Some of these approaches are extensions of similar approaches used for isotropic plate. However, a detailed study of stresses and strains distribution along the thickness in the central region of composite laminates has apparently never been presented. This paper will give the numerical stress and strain distributions in the symmetric composite laminates subjected to pure bending.

II. Analysis Model

The composite laminates considered herein consist of a number of perfectly bonding layers. Each layer is treated as a homogeneous orthotropic material. The geometry of the sample is shown in Fig. 1. Its elastic constants are:

$E_1 = 137.89\text{GPa}$

$E_2 = E_3 = 14.47\text{GPa}$

$\nu_{12} = \nu_{23} = \nu_{31} = 0.21$

$G_{12} = 5.86\text{GPa}$

$G_{23} = 0.5 \times E_2 / (1 + \nu_{23})$

For simplictity and without loss of generality, we restrict our attention to the cases of symmetric angle-ply and cross-ply laminates with ply thickness $h_0 = 6.35\text{cm}$. The composite laminate has a width $2b = 30.48\text{cm}$ and is subjected to a pure bending $\kappa_x = 0.002$.

According to the classical laminate theory (CLT), the stresses and strains may be obtained as follows:

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{pmatrix} \begin{Bmatrix} M_x \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^{0} \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z$$

and

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_i = \begin{pmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{pmatrix}_i \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

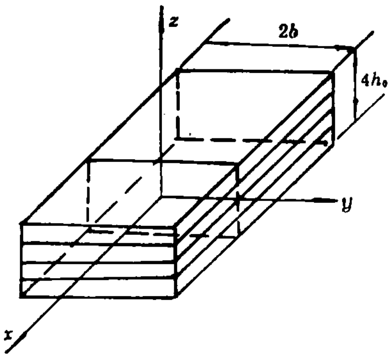


Fig. 1 The geometry of sample

in which the subscript i denotes the i-th layer. The interlaminar stresses σ_z, τ_{zx} and τ_{zy} are neglected in the classical laminate theory.

The composite laminate is sufficiently long that, in the region far away from the ends, end effects are negligible by virtue of Saint Venant's principle. Consequently, stresses and strains in the composite are independent of the x-axis. So, the length of sample in x-direction has

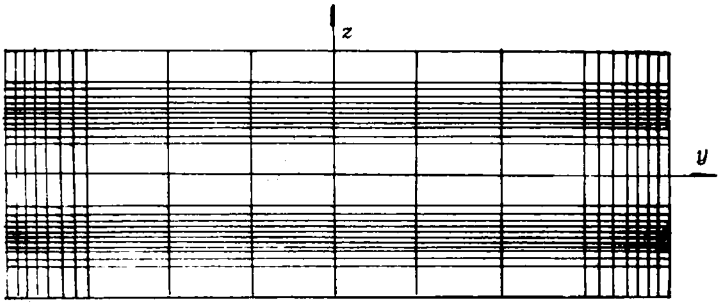


Fig. 2 The mesh configuration of sample

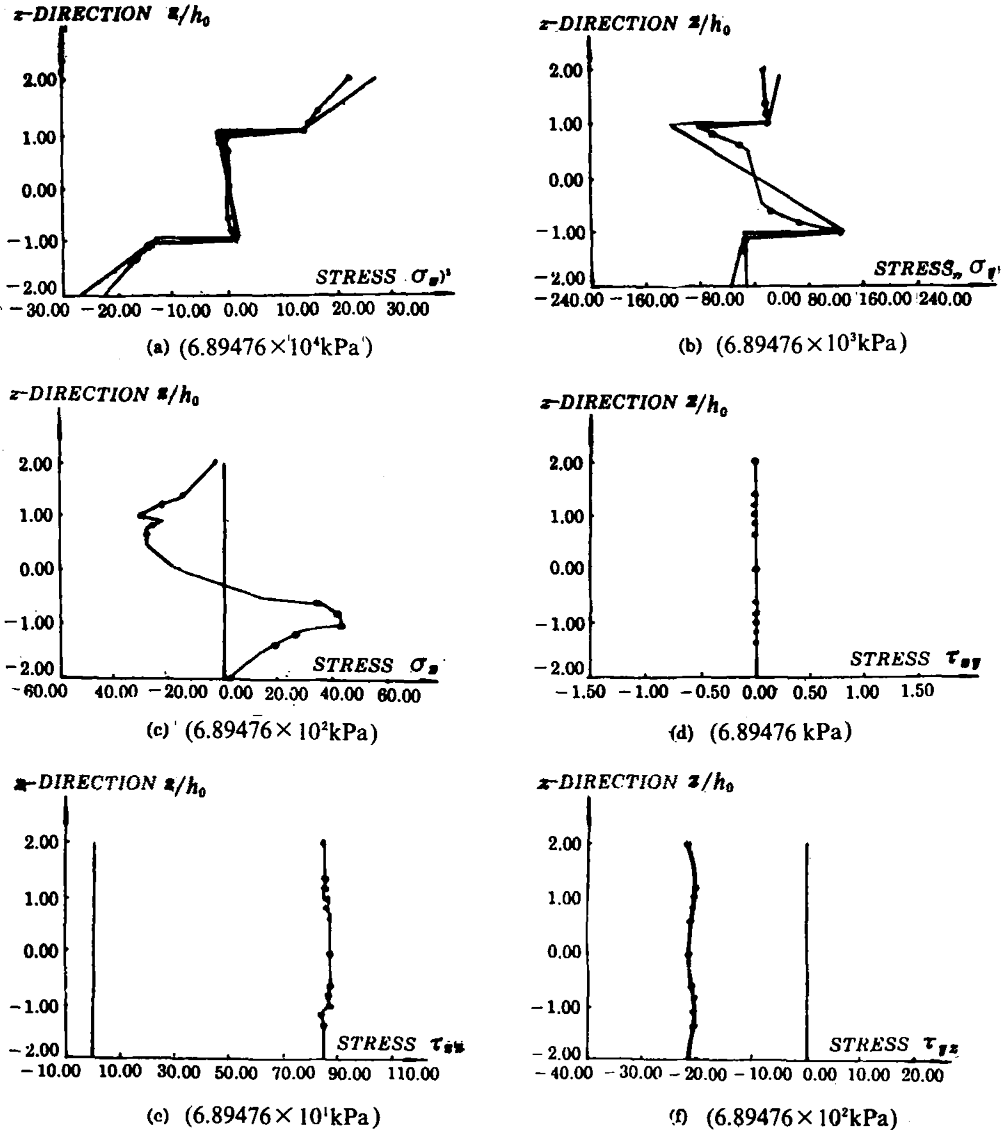
nothing to do with the stress analysis. The real length of calculated sample is $L=2.54$cm. The sample is divided into 480 3-D finite elements. There are 1050 nodes in the numerical model. The mesh configuration is shown in Fig. 2.

III. Numerical Stress and Strain Distribution

The present study focuses on the stress and strain distributions along the thickness in the central region of composite laminates. So, the numerical results of stress and strain distributions in the following figures are along the z-axis ($x=2.54$ and $y=0.0$).

The results for pure bending on the symmetric cross-ply laminate with $[0,90]_s$ fiber orientations are shown in Fig. 3 (a—f).

The classical laminate theory gives linearly distributed stresses σ_x and σ_y (the curves without dots) in each layer. The stresses $\sigma_z, \tau_{xy}, \tau_{xz}$ and τ_{yz} are equal to zero.



(a) (6.89476×10^4kPa)

(b) (6.89476×10^3kPa)

(c) (6.89476×10^2kPa)

(d) (6.89476 kPa)

(e) (6.89476×10^1kPa)

(f) (6.89476×10^2kPa)

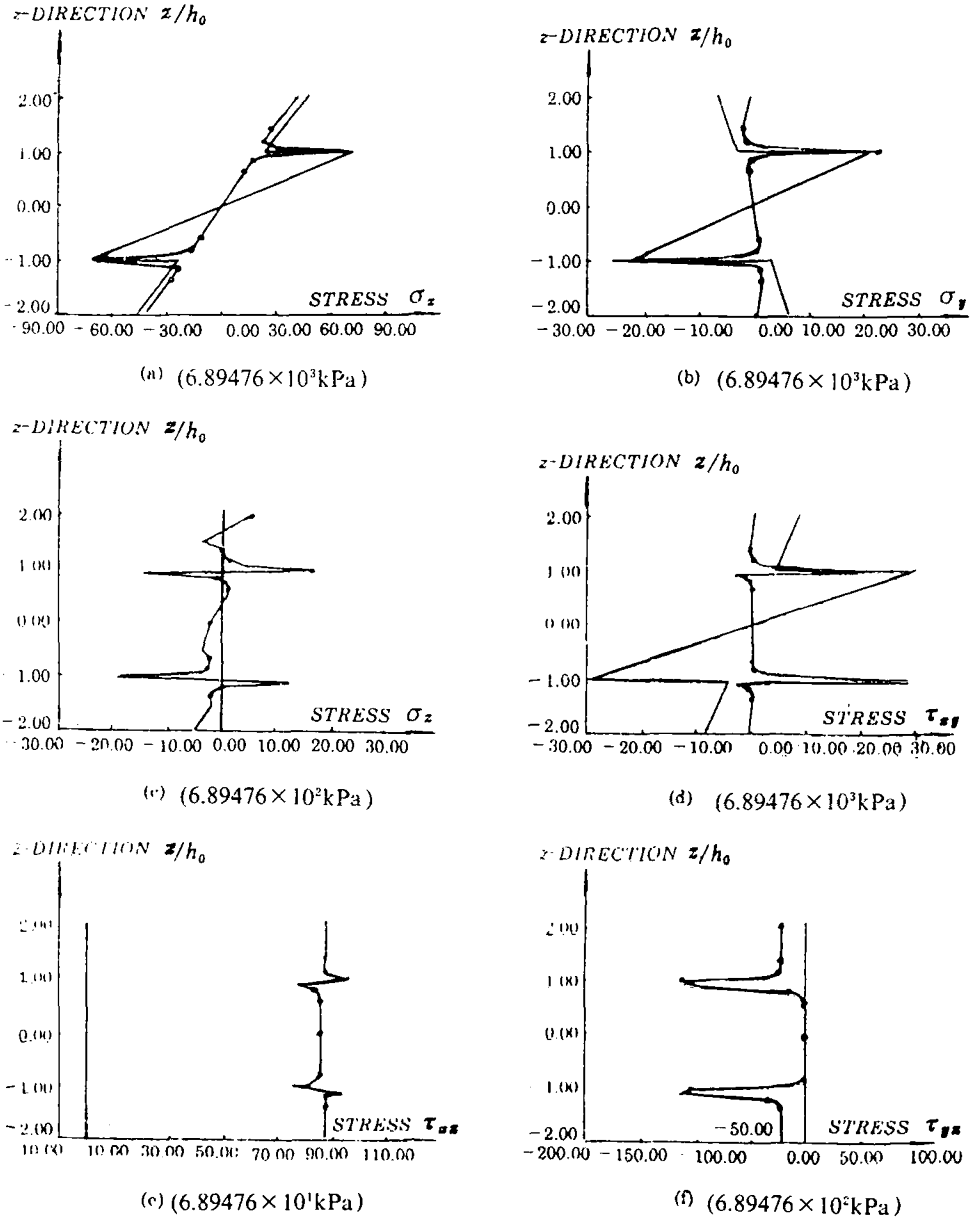
Fig. 3 The stress distribution of the cross-ply laminate under pure bending

The results of the numerical modal (the curves with dots) show that the distributions of main stressed σ_x and σ_y are similar to that calculated by the CLT. But the interlaminar stress σ_z is different from the result by the CLT. The value of stress σ_z is large because there is a bonding interface between adjacent layers. The stress τ_{xy} is equal to zero. The values of the stresses τ_{xz} and τ_{yz} are small.

The results of symmetric angle-ply laminate with the stacking sequence $[30, -30]_s$ subjected to the pure bending are shown in Fig. 4(a—f).

The classical laminate theory also gives linearly distributed stresses σ_x, σ_y and τ_{xy} in each layer. The interlaminar stresses σ_z, τ_{xz} and τ_{yz} are equal to zero.

It is shown that there is a strong concentration for all stresses near bonding interfaces



(a) (6.89476×10^3kPa) (b) (6.89476×10^3kPa)

(c) (6.89476×10^2kPa) (d) (6.89476×10^3kPa)

(e) (6.89476×10^1kPa) (f) (6.89476×10^2kPa)

Fig. 4 The stress distribution of angle-ply laminate $[30, -30]_s$ under pure bending

from the 3-D numerical model. The distribution of main stress σ_x is linear in each layer, but there is a concentration near the interfaces in the central region. The stresses σ_y and τ_{xy} are close to zero in each layer, but they are very large near bonding interfaces. It is quite different from the results by the classical laminate theory. The interlaminar stress σ_z and τ_{yz} also have a strong concentration near the interfaces. Although the stress τ_{xz} is smaller than the main stress σ_x, the stress τ_{xz} also has a concentration near it. So, the angle-ply laminates are in 3-D stress state near interfaces, although they are close to 2-D stress state in each layer.

The strain distributions of angle-ply laminate with $[30, -30]_s$ fiber orientations are shown in Fig. 5 (a--f).

The numerical results present a linearly distributed strains ε_x, ε_y and γ_{xy} in each layer, which are close to assumption of the classical laminate theory. But the strains distributions are different from the results of the classical laminate theory near bonding interfaces. There is an

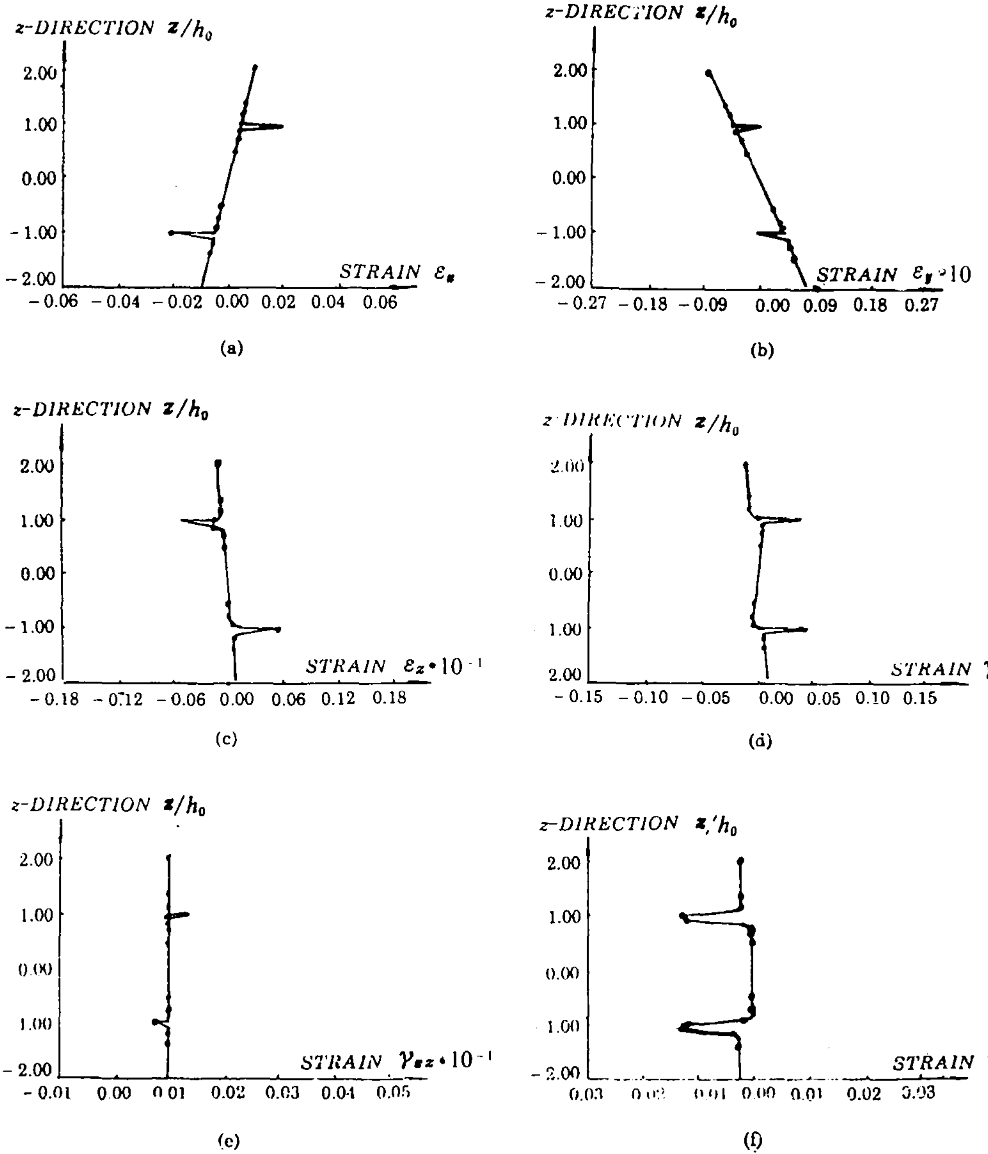


Fig. 5 The strain distribution of the angle-ply laminate $[30, -30]_s$ under pure bending

abrupt change of all strains near interfaces. The strains ε_z, γ_{xz} and γ_{yx} are nonzero. The Kirchhoff-Love hypotheses of straight inextensional normals for the entire plate can not present sufficiently accurate results for the symmetric angle-ply laminates.

IV. Conclusion

The stress and strain distributions are studied with an emphasis on the interlaminar effect in the central region of symmetric composite laminates. It has been pointed out that the numerical distributions of stresses in cross-ply laminate is coincident with the results by the classical laminate theory. The CLT yields sufficiently accurate results for cross-ply laminates. But the angle-ply laminates are different from the cross-ply laminates. The adjacent $\pm\theta$ layers lead to high interlaminar stresses, an abrupt change of all stresses and strains, and a strong concentration for stresses near bonding interfaces. The existence of bonding interface makes the symmetric angle-ply laminates being in 3-D stress state. This kind of the interlaminar effect in the central region of composite laminates has not been taken into consideration in the classical laminate theory and other models such as 2-D shear-flexible models. The error of the strain assumption of several approaches is very large for symmetric angle-ply laminates.

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